Frailty Models for Commercial Mortgages

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1 Introduction

Credit risk affects virtually all aspects of financial activities. It is an important factor in pricing financial products and has profound influence on risk management. Moreover, policy makers and regulators pay special attention to credit risk when they design economic policies and regulatory frameworks.

There is an extensive literature on the measurement and management of credit risk. Researchers have used various approaches to model the credit risk of corporate debt, mortgages and derivatives. In general, these models can be divided into two categories – structural models and reduced-form models. This article presents a reduced-form model that contains frailty factors to predict commercial mortgage default.

Reduced-form models rely on two sources of information to explain credit risk. One source is the financial information of borrowers, which is used to track the idiosyncratic part of the credit risk. The other source comes from macro variables, which approximates the systematic part. Failure to explain all the systematic risk would introduce biases when estimating risk measures. Das et al. (2007) – while studying corporate bond defaults – provided evidence that macro variables alone were not enough to explain all the systematic risk. They further demonstrated that the lack of explanatory variables underestimates value-at-risk. To produce unbiased estimates, they proposed frailty models for corporate bond credit risk, to account for the unexplained part of systematic risk.

Frailty models can be classified into two types - using a characterization put forward by Cox et al. 1981: parameter-driven models and observation-driven models. Both types of models have been used to predict credit risk. For example, parameter-driven models have been used to track the credit risk of corporate debts in Duffie et al. (2009) and Koopman and Lucas (2008), and to forecast mortgage default in Kau, Keenan and Li (2011). Meanwhile, using observation-driven models, Creal, Koopman and Lucas (2013) and Creal et al. (2014) investigated corporate defaults. The predictability of latent factors is a main feature of observation-driven models. This feature indicates that current period frailty factors can be computed using only past information. In contrast, the computation of frailty factors in parameter-driven models not only requires past information but also future and current information. Inference of parameter-driven models therefore generally requires simulation, which is time consuming with large data sets. The estimation of observation-driven models is in comparison rather straightforward.

In this paper, we develop a novel framework to model systematic risk of mortgages. Specifically, we match default rates in multiple dimensions by extending the generalized autoregressive score (GAS) models proposed in Creal, Koopman and Lucas (2013). Our data consists of commercial mortgages in the U.S. retail market from 1997 to 2013. We construct a series of models and use multiple tests to demonstrate the advantages of our framework.

To the best of our knowledge, this is the first paper that uses observation-driven models to predict mortgage defaults. We show that the new class of models we propose has better tractability compared with parameter-driven models. For instance, although our dataset has more than two million records, and our most complex model incorporates up to 15 frailty factors, the estimation process only takes two minutes using a standard desktop computer. Compare this with for example Kau, Keenan and Smurov (2006) who use parameter-driven models to predict mortgage defaults using a small data set. Their method requires simulations and is very time consuming and therefore practically infeasible when using large data sets typically encountered in practice.

In addition to the merit of tractability, our multi-factor formulation is able to match the default rates of mortgages in multiple dimensions. While a single latent factor suffices to match the temporal fluctuation of default rates, it may not capture the variation of default rates along other dimensions. For example, our empirical analysis provides evidence that the origination month and the originator preference influence the default rates of commercial mortgages. To model the influence of these variables, we group mortgages by these two variables and allow latent factors to vary by groups. Compared with GAS models using a single factor, our multi-factor frailty models feature improved empirical fits.

The remainder of this paper is organized as follows. The next section reviews the literature. Section 3 formulates our dynamic frailty models. In section 4, we discuss empirical applications. The last section offers some concluding remarks.

2 Literature Review

To motivate our theoretical framework and demonstrate the merit of frailty models, it is useful to review the literature on credit risk modeling. We first review the studies that used either structural or reduced-form approaches to model the credit risk of corporate debt. Next, we focus on two types of reduced-form models augmented by frailty factors: observation-driven models and parameter-driven models. The bulk of the literature focuses on corporate debt - due to the fact that such data is most readily available. Since our paper deals with commercial mortgages, we review the relatively small literature covering the current state of the art models for mortgage credit risks.

Research on credit risk prediction has a long history, dating back to Beaver (1968) and Altman (1968). Since then, numerous models have been developed to assess risk factors and predict default events. Following the classification in Altman et al. (2005), credit risk models can be categorized into two groups: structural-form models and reduced-form models. A majority of structural-form models are based on the framework proposed by Black and Scholes (1973) and Merton (1974), which utilizes the principle of option pricing. This principle assumes that a corporation will default when its assets drop to a sufficiently low level relative to its liabilities. As a result, all the relevant credit risk elements are functions of structural characteristics of the corporation, such as asset volatility and capital structure. Similar structural-form models have been proposed in Black and Cox (1976), Jones, Mason and Rosenfeld (1984), Fischer, Heinkel and Zechner (1989), Shimko, Tejima and Van Deventer (1993), Leland (1994), Longstaff and Schwartz (1995), Nielsen, Saà-Requejo and Santa-Clara (2001), and Hui, Lo and Tsang (2003). Both macro variables and financial information of borrowers have been used in these models.[1]

While structural-form models assess credit risk elements such as probability of default and recovery rates through an implied process of the value of a corporation, reduced-form models impose separate and explicit assumptions on default probabilities and recovery rates. Specifically, reduced-form models assume credit risk elements not only relate to the structural features of the firm, but also depend on macro variables and financial information of borrowers. This line of research started with Beaver (1968) and Altman (1968) who used discriminant analysis as the main tool.[2] Since Ohlson (1980) and Zmijewski (1984), binary response models like logit and probit regressions have been introduced into credit risk modeling Most of these models estimate single-period default probabilities or credit scores. Some recent studies began

[1] Structural-form models have also been widely used by financial corporations, such as Moody's KMV, a leading provider of quantitative credit analysis tools. For a complete description of Moody's KMV method, see Crosbie and Bohn (2003).

[2] These models are often called "the first generation of reduced-form models" (Duan, Sun and Wang 2012).

to extend the prediction horizon to multiple periods using multiple logit models (see for example, Campbell, Hilscher and Szilagyi 2008).

All the reduced-form models mentioned above are based on the assumption that macro variables are enough to explain all the systematic risk. However, Das et al. (2007) provide evidence that this assumption can be violated. In particular, Duffie et al. (2009) introduce the notion of common latent factors - or so called frailty factors - into the default intensity of their proportional hazard models for corporate debt credit risk. They showed that failure to control for these latent factors caused downward biases in calculating value-at-risk. In similar contexts, Duan and Fulop (2013) proposed frailty models with forward intensity methods and attempted to overcome the potential computational burden induced by models in Duffie (2009).

Koopman and Lucas (2008) considered frailty methods with formulations other than proportional hazard models. In particular, they added latent dynamic factors to logistic regressions and developed a non-Gaussian multivariate state-space model to predict corporate default. Koopman, Lucas and Schwaab (2012) extend the framework of Koopman and Lucas (2008) by jointly modeling macro variables with default events, and also include industry effects. Schwaab, Koopman and Lucas (2015) consider a framework similar to the one in Koopman, Lucas and Schwaab (2012) adding regional effects to model default outcomes. All the models involving latent factors discussed so far are parameter-driven models. Estimation of such models generally involves simulation-based algorithms, such as particle filtering and importance sampling. Typical applications of these algorithms are time consuming and therefore impractical to implement with large data sets.

Observation-driven models have also been used to model credit risk. In particular, Creal, Koopman and Lucas (2013) designed a class of observation-driven models - called GAS models - and applied them to Moody's credit rating data. GAS models use scaled scores of the likelihood function to update the dynamics of latent factors. Using these models, Creal et al. (2014) jointly model macro variables and default outcomes with data of mixed-measurement and mixed-frequency.[3]

[3]If one applies GAS models to binary logistic regressions and chooses the intercept as the dynamic parameter, then score of the dynamic intercept is actually the generalized residual defined in Gouriéroux et al. (1987). In this context, GAS models reduce to the generalized autoregressive moving average models proposed in Shephard (1995) and Benjamin, Rigby and Stasinopoulos (2003).

While the bulk of the literature covers corporate debt credit risk, there are a few papers studying mortgage defaults. Frailty factors play an important role in modeling mortgages and are called “baseline hazards” in the literature. Currently, non-parametric methods are used in modeling frailty factors. Among all non-parametric methods, the flexible methods proposed in Han and Hausman (1990), Sueyoshi (1992) and McCall (1996) are popular choices (for an application, see Deng, Quigley and Order 2000). Shared frailty, used in Follain, Ondrich and Sinha (1997), is an alternative to flexible methods. Kau, Keenan and Li (2011) extended Follain, Ondrich and Sinha (1997) by considering regional effects. Finally, Kau, Keenan and Smurov (2004) and Kau, Keenan and Smurov (2006) used parameter-driven models to capture baseline hazards of residential mortgages.

3 Model Formulation

We develop in this section a novel class of models for systematic credit risk of mortgages, extending the generalized autoregressive score (GAS) models proposed in Creal, Koopman and Lucas (2013).

To set up the model, consider a set of n_t mortgages and let the vector of default status be denoted as $y_t = [y_{1,t}, ..., y_{n_t,t}]$, where n_t is the number of mortgages at time t. The element y_{it} of this vector, is a binary variable referring to the default status of mortgage i at time t; it equals to one if a default happens at time t, and zero if the borrower can make timely payment. Let us further denote the default probability as $\pi_{i,t}$, i.e. $\pi_{i,t} = P(y_{i,t} = 1)$. To relate the default probability with covariates, we adopt a binary logit model:

$$\pi_{i,t} = exp(\mu_{i,t})/(1 + exp(\mu_{i,t}))$$

where $\mu_{i,t}$ is transformed default probability. It is defined as a linear combination of covariates and a latent factor:

$$\mu_{i,t} = x'_{i,t}\beta + f_{i,t}.$$

where the vector $x_{i,t}$ consists of a series of observed variables, such as financial information of the mortgage, β is a coefficient vector and constant across all mortgages, and $f_{i,t}$ is a latent factor or factors - as clarified later. It is important to emphasize one interpretation of the above equation. The presence of $f_{i,t}$ can be viewed as the *intercept* to the default intensity. Hence, a latent factor or set of factors affect the baseline defaults. More specifically, because fixed income products have different credit quality, their exposure to common sources of systematic risk varies. To model this feature, researchers usually group fixed-income products according to certain criteria, and allow latent factors to vary across different groups. Rating classes are a popular criterion in grouping corporate debts. For example, Creal et al. (2014) grouped corporate debts by rating classes, and their models have three factors based on multiple classes. Schwaab, Koopman and Lucas (2015) grouped corporate debts not only by rating class, but also by region and industry. However, the grouping criteria are not obvious for mortgages, as there is no rating class for mortgages. Researchers have to define their own criteria. For instance, in Kau, Keenan and Li (2011), the authors use locations of mortgages as the criterion to define latent factors.

Our empirical analysis suggests that mortgage default rates vary with origination month and originator preference. Based on this evidence, we choose these two properties of mortgages as our grouping criteria. Specifically, we group mortgages of consecutive origination months in the same group and use c_i^1 to denote the group number of mortgage i according to origination month. Meanwhile, mortgages of similar originator preference are grouped together and c_i^2 represents the group number of mortgage i according to the grouping criterion of originator.

We assume a common latent factor for mortgages in the same group and allow latent factors to vary across different groups. Since mortgages in different groups have distinct latent factors, every grouping criterion corresponds to a separate set of latent factors. Combining this assumption with our two grouping criteria, we decompose $f_{i,t}$, the latent factor specification for mortgage i at time t, into two parts:

$$f_{i,t} = f^1_{c_i^1,t} + f^2_{c_i^2,t}$$

where $f^1_{c_i^1,t}$ represents the set of frailty factors related to "origination month", and $f^2_{c_i^2,t}$ stands for the set of frailty factors relevant to "originator preference". Generally, if n grouping criteria are identified then $f_{i,t}$ is decomposed into n parts.

We further assume that the latent factors have an autoregressive form, namely:

$$f^k_{c_i^k,t} = \theta_k f^k_{c_i^k,t-1} + \alpha_k s^k_{c_i^k,t-1}, k \in \{1, 2\}$$

The time subscript to the innovation $s^k_{c_i^k,t-1}$ indicates that it is computed using time $t-1$ information. The choice of innovation is crucial in updating the dynamics of frailty factors, and use generalized residuals, as dubbed by Gouriéroux et al. (1987). Specifically, we characterize the innovations as:

$$
\begin{aligned}
s^k_{c^k_i,t-1} &= \bar{y}^k_{c^k_i,t-1} - \hat{y}^k_{c^k_i,t-1} \\
\bar{y}^k_{c^k_i,t-1} &= \sum_{j=1}^{n_{t-1}} y_{j,t-1} 1_{c^k_j = c^k_i} \\
\hat{y}^k_{c^k_i,t-1} &= \sum_{j=1}^{n_{t-1}} \hat{\pi}_{j,t-1} 1_{c^k_j = c^k_i}
\end{aligned}
$$

where $\bar{y}^k_{c^k_i,t-1}$ is the empirical default rate for group c^k_i at time $t-1$, and $\hat{y}^k_{c^k_i,t-1}$ is the fitted default rate for group c^k_i at time $t-1$. $1_{c^k_j=c^k_i}$ is an indicator variable; it equals to one if mortgage j is in group c^k_i and zero otherwise. Using this indicator variable, we include only information from group c^k_i to update the frailty factors related to mortgage i. Likewise, $\hat{\pi}_{j,t-1}$ is the estimated default probability (PD) for mortgage j. Intuitively, our innovation term is the difference between empirical default probability and fitted default probability. The innovation therefore measures the distance between models and data. If the innovation term is positive/negative, then the empirical PD is larger/smaller than fitted one.

Finally, it is important to note that the coefficients of the frailty factors, namely θ_k and α_k, are the same across all groups. This restriction enables us to keep the model parsimonious and tractable. Practically speaking, our data set is large and we have more than 15 groups to characterize the heterogeneity across mortgages. Without this restriction, the model would become intractable, and more than 30 parameters need to be estimated in a non-linear setting. In our model, the sign of the coefficient α_k determines the properties of frailty factor. A positive sign implies that the fitted PD will be adjusted towards the empirical one whenever there exists a mismatch between the model default predictions and the data.

4 Empirical Applications

4.1 Data and Variables

Our mortgage data contains 2,207,588 records of commercial mortgages in the U.S. retail market. The records start in July 1996 and end in the middle of 2013, with none of the mortgage pre-dating the start of the sample. There are 7479 distinct mortgages and all of them are 10-year-balloon mortgages.

As discussed earlier, default risk of mortgages consists of idiosyncratic risk and systematic risk. To explain idiosyncratic risk, we use mortgage age, debt service coverage ratio, and an indicator variable reflecting servicers' warning. For systematic risk, we use lagged values of default rates and a single frailty factor or multiple frailty factors, such as origination month frailty and originator frailty, to track the temporal fluctuation of default rates. In the following paragraphs, we define each of these variables and discuss their influence on default risk.

Mortgage age is defined as the number of months passed since the initiation of mortgages. It has been widely used in the literature to explain the trend of mortgage default. Most of the papers we reviewed discussed only fully-amortizing mortgages, which have zero balance at maturity date. Since the payment due at maturity is small compared to the principal, the default barely happens at maturity. However, our mortgages are 10-year-balloon mortgages, which are partially-amortizing. That is, the borrowers need to make a balloon payment at maturity, which is relatively large compared to the principal. Generally, this payment is funded by refinancing. Due to the potential failure of refinancing, borrowers of balloon mortgages are more likely to default at the maturity date than those of fully-amortizing mortgages.

Figure 1 provides evidence of high default rates at maturity for balloon mortgages. In the picture, default rate rises dramatically from the 120^{th} month (maturity month), peaks at the level of 0.03 in the 123^{th} month, then decreases sharply right after. This peak clearly shows the influence of balloon payment on default rates. Default could also happen before maturity because borrowers may not be able to make monthly payments. Figure 1 also demonstrates this phenomenon by showing that the number of defaults gradually increases in the first 56 months and then declines until the maturity date. In light of these two peaks on the curve, we design a piecewise linear function to capture the influence of mortgage age on default rates. This function is called the age function and has the following form:

$$\begin{aligned}
age_1 &= min(age, 56) \\
age_2 &= max(min(120-56, age-56)) \\
age_3 &= min(max(age-120, 0), 3) \\
age_4 &= max(0, age-123) \\
Age_function &= age_1\beta_1 + age_2\beta_2 + age_3\beta_3 + age_4\beta_4
\end{aligned}$$

where age is short for mortgage age in the formulas above.

Debt Service Coverage Ratio (DSCR) is another crucial variable in modeling mortgage default. It is defined as the ratio between net operating income and current debt obligations. DSCR larger one indicates borrowers have enough cash flow to make monthly payments. Otherwise, borrowers may default. We use the original value of DSCR without any transformation.

We use a dummy variable indicating whether servicers pay special attention to mortgages. We name this indicator variable as SW. If SW is one, then the servicer may consider the mortgage at high default risk. Otherwise, the situation of the mortgage is normal. SW enters into our models without transformations.

After describing variables for idiosyncratic risk, we turn our attention to variables tracking systematic risk. The first variable we use is the lagged value of default rates, denoted as lagged_PD. Since our models divides mortgage data into several groups, we compute lagged_PD for each group. We use logit transformation of lagged_PD here, because logistic regression is utilized in our models. The transformed variable is defined in the following way:

$$lagged\_PD' = log((lagged\_PD)/(1 - lagged\_PD))$$

Our empirical analysis shows this transformation significantly improves our model performance, so we use $lagged\_PD'$ instead of the original values.

The second set of variables we use are frailty factors, which capture the unexplained part of systematic risk. We develop three formulations for frailty factors. These formulations differ in the criterion to group the mortgages, and therefore, in the number of frailty factors. Since we use data of monthly frequency, all of these frailty factors are updated every month. The first formulation is called “single frailty”. In this formulation, all mortgages belong to one group, which indicates a common factor for all mortgages. The second formulation is called “origination frailty’, in which we divide mortgage data into several groups by their origination months. Accordingly, we have a separate frailty factor for each origination group. The third formulation is called “originator frailty”, where we group mortgages by their originators and obtain a multi-factor formulation.

4.2 Estimation

We build six models to examine the influence of the variables specified in the previous subsection on mortgage default rates. Table 1 lists the variables used in each model. The first three models, Static I, II and III, only contain static variables, such as $Age\_function$ and $DSCR$. The next three models are dynamic models which include both static variables and various frailty factors. In Table 1 the models appear from specific to general, i.e. each model is nested in the model on its right. For example, Static I is nested in Static II, because the former uses original values of age while the latter uses a flexible age function. Similarly, Dynamic III includes both origination frailty and originator frailty, while Dynamic II only contains origination frailty.

For the dynamic models, grouping of mortgages is a key step in constructing frailty factors. For "single frailty", no grouping is needed. However, when using "origination frailty" and "originator frailty", we need to carefully consider the groupings. On the one hand, a small group size is desired to ensure the similarity in credit quality among mortgages. On the other hand, a group that is too small may produce imprecise estimates, since mortgage default is a rare event. The group size should be large enough to produce smooth estimates of empirical default rates, which are inputs to our frailty factors. Therefore, to pick a proper group size, we have to strike a balance between controlling biases and producing smooth estimates.

For origination frailty, we choose two years as our window size to group mortgages. Ideally, mortgages originated in the same month should form a group, because mortgages initiated in the same month have less variation in credit quality than mortgages initiated in a relatively long period. However, the number of mortgages initiated in one month is too small to produce a smooth estimate of default rates. The two-year window size is chosen based on empirical tests. By comparing a number of alternative window sizes from six months to four years, we find that the window size of six months produces non-smooth default curves, and the window size of four years groups mortgages of different qualities together. The two-year window size appears to strike a balance as it not only controls the variation in mortgage quality but also keeps estimates of default rates smooth.

For originator frailty, we group mortgages based on the performance of their originators in the past. For mortgages of each originator, we calculate the differences between empirical default rates and fitted default rates, the later implied by Dynamic II. Using these differences, we divide originators into 6 groups. We denote the difference as Δ and display the grouping criterion in Table 2. This criterion has limitations, as it relies on prediction results from other models. When more information is available, one could potentially design alternatives to assess the preference of originators.

We assume conditional independence for dependent variables in all the models and use standard maximum likelihood methods to estimate parameters. The data set has more than two million records, and the most sophisticated model has more than 15 factors which require updating every month. To accelerate the estimation process, we derive analytic gradients and to obtain good initial values for the optimization, we estimate the models sequentially from specific to general. As the models are nested, we select the initial values for the estimation based on the final estimates of the restricted model. Following these procedures, we can complete the estimation for our most complex model in two minutes with a standard desktop computer.

4.3 Results

Table 3 reports the estimation results for both static and dynamic models. All parameters are highly significant and have the expected signs. Except for the Static I model, we use a piecewise-linear function to capture the influence of mortgage age on default rates in all models. In particular, since our data set consists of balloon mortgages, a piece-wise linear function is designed to accommodate the effects of balloon payments on mortgage default. We find that the coefficients of age_1 and age_3 are positive, and the coefficients of age_2 and age_4 are negative. This is consistent with the trend of empirical default rates shown in Figure 1. Our results provide evidence that the effect of DSCR on mortgage default is negative as expected, since borrowers with higher income are less likely to default. Note that the coefficient of $lagged\_PD'$ reduces from 1.119 in static model III to approximately 0.7 in dynamic models. This reduction implies the explanatory power of the latent factors for systematic risk. Because $lagged\_PD'$ is a proxy for the systematic variation of PD, the parameter decreases indicate that part of its loadings to explain systematic variation transfer to the latent factors we added in dynamic models.

The lower part of Table 3 presents parameter estimates related to latent factors. Note that Dynamic III has four parameters, while Dynamic I and II only have two parameters. In Dynamic I, α and θ are the autoregressive and innovation coefficients of the single-frailty factor. In dynamic II and III, α_1 and θ_1 are coefficients for the origination factors, and α_2 and θ_2 are coefficients for the originator factors. The positive estimates of α, α_1, and α_2 indicate that fitted default rates are adjusted towards empirical default rates when there are mismatches between these two rates.

The last row of Table 3 reports the log-likelihood of each model. As we can see, this number increases from the leftmost model to the rightmost model, and the smallest difference between any two likelihoods is over 1000. Since Dynamic III has the highest log-likelihood and the first five models are nested in it, these numbers suggest Dynamic III has the best sample fit. In the following paragraphs, we use a series of diagnostic plots to demonstrate the advantages of Dynamic III.

To begin with, Figure 2 illustrates the improvement from Static I to Static II. As we know, Static II augments Static I by introducing a piecewise-linear age function. In Figure 2, the empirical default rates by mortgage age are represented by a solid line. The default curve implied by Static II, shown by a dark dashed line, follows the empirical default curve closely. By contrast, the default curve generated by Static I, drawn by a light dashed line, barely tracks the trend of the empirical default curve. This comparison is not surprising at all, because a constant coefficient cannot fit the non-linear effect of age on default.

While Figure 2 shows the advantages of the age function, Figure 3 and 4 demonstrate the benefits of tracking systematic risk with $lagged\_PD'$. In these two figures, default rates are computed by exposure months, and the empirical default curve exhibits significant default clustering around 2005 and 2011. Unfortunately, as shown in Figure 3, the curves generated by Static I and II cannot approximate the default clusters. In Figure 4 the curve implied by Static III roughly follows the empirical default curve and its clustering patterns. These figures convey that $lagged\_PD'$ is a good proxy for systematic risk. However, the mismatches between the empirical curve and the fitted curve are still not negligible.

Considering the mismatches in Figure 4, we further include single-frailty in Dynamic I. Namely, we allow a separate dynamic intercept for each month, and the intercept is the same for all mortgages. Figure 5 displays the fitted curve by Dynamic I along with the empirical curve. The fitted curve matches the empirical one not only in the period of default clustering (2005 and 2011), but also in almost all other periods of the data set. The closeness of these two curves shows that mismatches are largely corrected by the dynamic intercept.

These results convey that a dynamic intercept would suffice if we only want to track the default rates in the dimension of exposure month. However, inconsistency appears again when we investigate whether fitted default rates follow empirical ones along the dimension of origination month. To show this inconsistency, we group mortgages by their origination months and plot default rates in Figure 6. We observe a noticeable gap for mortgages initiated after 2008. While the empirical default rate drops to zero for these mortgages, the fitted default rate increases until 2011 and remains above 0.008 in the next two years. Besides, the mismatches for mortgages initiated before 2004 are not negligible, either. Clearly, the model with a single dynamic intercept overestimates the default risk for mortgages initiated after 2008 and yields rough predictions for mortgages initiated before 2004.

To narrow the gaps in Figure 6, we allow separate intercepts for different origination groups in Dynamic II. Specifically, we group mortgages by origination with a two-year window size and allow each group to have its own intercept (the details appeared in the model specification section). Figure 7 evidently shows the improvement after allowing a separate intercept for each origination group. The fitted default rates by Dynamic II sharply decline after 2008 and remains at zero thereafter. Additionally, the fitted curve also closely tracks the empirical one before 2004.

In Dynamic II, we fit default rates along dimensions of not only exposure month but also origination month. As known, originator preference also influences default rates significantly. We proceed to explore this fitting dimension and report our findings in Figure 8 and 9. In Figure 8, we re-group the data by originator group and plot the empirical curve with the fitted curve of Dynamic II. The gap between these two curves illustrates that Dynamic II cannot model the variation of default rates caused by originator preference.

Although Figure 8 shows mismatches of default rates at the level of originator group, it is still not clear whether there are systematic mismatches within each group. To examine the possibility of systematic deviation, we divide each originator group into several sub-groups by mortgage age, and plot the fitted and empirical rates in Figure 9. Since each group corresponds to an empirical curve and a fitted curve, we plot 12 curves in Figure 9. These 12 curves confirm the existence of systematic deviation within each originator group. For the first three groups, these curves imply that the fitted rates are lower than the empirical rates for almost all mortgage ages. For the last three groups, these curves suggest an over-estimation of default rates across nearly all mortgage ages.

Motivated by Figure 8 and 9, we supplement origination frailty factors with originator frailty factors in Dynamic III. Figure 10 and 11 support our modification. In Figure 10, the empirical curve largely overlaps the fitted curve by Dynamic III. This overlapping shows the reduction of gaps by using extra frailty factors. Furthermore, Figure 11 plots six sets of overlapping curves. This indicates that the systematic deviations shown in Figure 9 are also corrected.

5 Conclusions

To the best of our knowledge we are the first to use observation-driven frailty models to predict commercial real estate mortgage defaults. In particular, we introduced a class of frailty models to track the variations of mortgage default rates in multiple dimensions. Our frailty factors track origination and originator characteristics. The frailty factors enable our models to track the variation of default rates in three dimensions: exposure month, origination month and originator group. In our empirical application, a series of models were constructed to investigate the effects of frailty factors on default rates. We tested performance of the models using a data set with more than two million records. The models exhibit computational advantages when using the large data set. Since our data set consists of balloon mortgages, a piecewise linear function is designed to accommodate the effects of balloon payments on mortgage default. Financial information of borrowers is also used to track idiosyncratic risk of mortgages. Diagnostic plots and statistic test results demonstrate the superior performance of our dynamic models. While we only explore three fitting dimensions, our framework can be easily generalized by including more latent factors and fitting default rates in more dimensions.

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Table 1: Components of Static and Dynamic Models

	Static I	Static II	Static III	Dynamic I	Dynamic II	Dynamic III
Age	X					
$Age\_function$		X	X	X	X	X
$DSCR$	X	X	X	X	X	X
SW	X	X	X	X	X	X
$Lagged\_PD'$			X	X	X	X
$Exposure\ Frailty$				X		
$Origination\ Frailty$					X	X
$Originator\ Frailty$						X

Notes: This table describes components of each models in empirical applications. X means the variable is a part of the model.

Table 2: The Grouping Criterion for Originator Frailty

Group	Difference
1	$\Delta > 0.02$
2	$0.01 < \Delta \leq 0.02$
3	$0 < \Delta \leq 0.01$
4	$-0.005 < \Delta \leq 0$
5	$-0.01 < \Delta \leq -0.005$
6	$\Delta \leq -0.01$

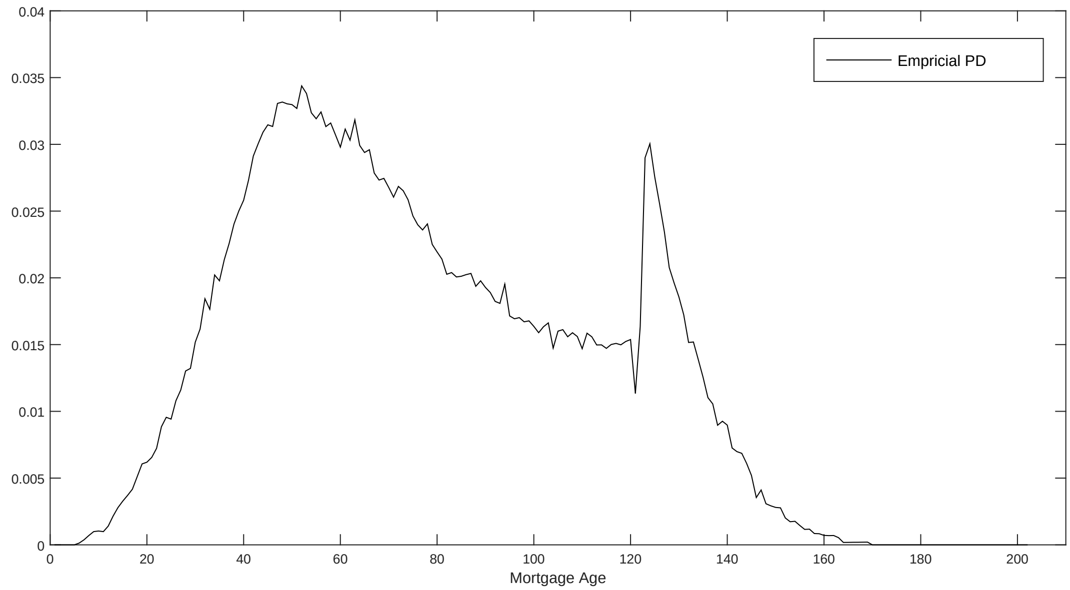
Notes: The table specifies the thresholds used in grouping originators.

Table 3: Estimates for Static and Dynamic Models

	Static I	Static II	Static III	Dynamic I	Dynamic II	Dynamic III
Age	0.0014	-	-	-	-	-
	(2.8)					
Age_1	-	0.0585	0.0429	0.0466	0.0407	0.0256
		(56)	(48)	(60)	(38)	(32)
Age_2	-	-0.017	-0.0178	-0.0207	-0.0207	-0.0220
		(-37)	(-50)	(-55)	(-45)	(-38)
Age_3	-	0.4965	0.4447	0.4253	0.362	0.3577
		(52)	(65)	(74)	(40)	(35)
Age_4	-	-0.1014	-0.1128	-0.1129	-0.1032	-0.1018
		(-51)	(-39)	(-34)	(-28)	(-33)
$DSCR$	-1.673	-1.628	-1.553	-1.530	-1.499	-1.491
	(-190)	(-157)	(-126)	(-139)	(-114)	(-120)
SW	-3.32	-3.2176	-3.29	-3.324	-3.329	-3.329
	(-147)	(-83)	(-77)	(-99)	(-80)	(-88)
$Lagged\_PD'$	-	-	1.119	0.7143	0.7881	0.7576
			(117)	(17)	(44)	(81)
$Intercept$	-1.7083	-3.86	1.37	-	-	-
	(-16)	(-8)	(-7.5)			
α	-	-	-	49.5	-	-
				(28)		
θ	-	-	-	0.992	-	-
				(672)		
α_1	-	-	-	-	28.9	20.21
					(117)	(86)
θ_1	-	-	-	-	0.987	1.02
					(1320)	(803)
α_2	-	-	-	-	-	7.13
						(92)
θ_2	-	-	-	-	-	0.995
						(772)
$logL$	-164524	-156510	-148925	-148137	-146523	-145392

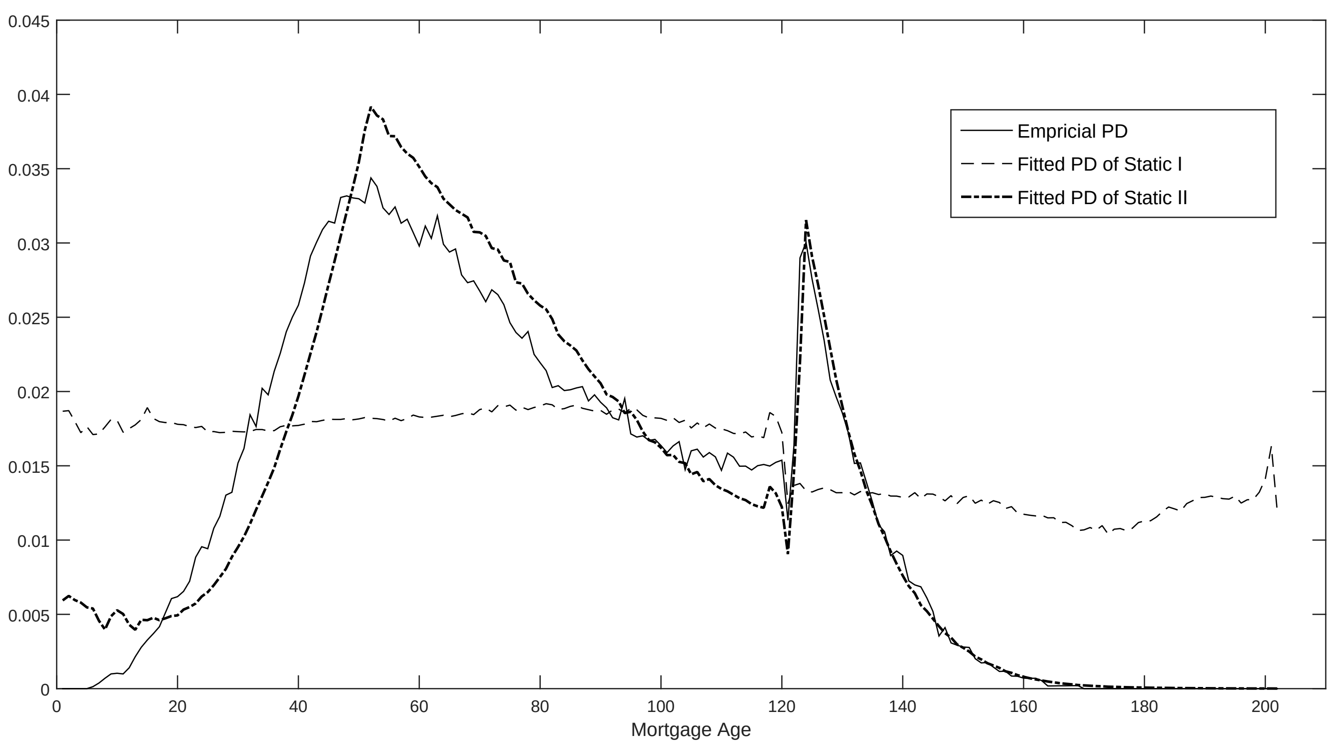
Notes: This table reports the estimates for static and dynamic models. T-statistics are in parentheses.

Figure 1: Empirical Default Rates (PD) by Mortgage Age



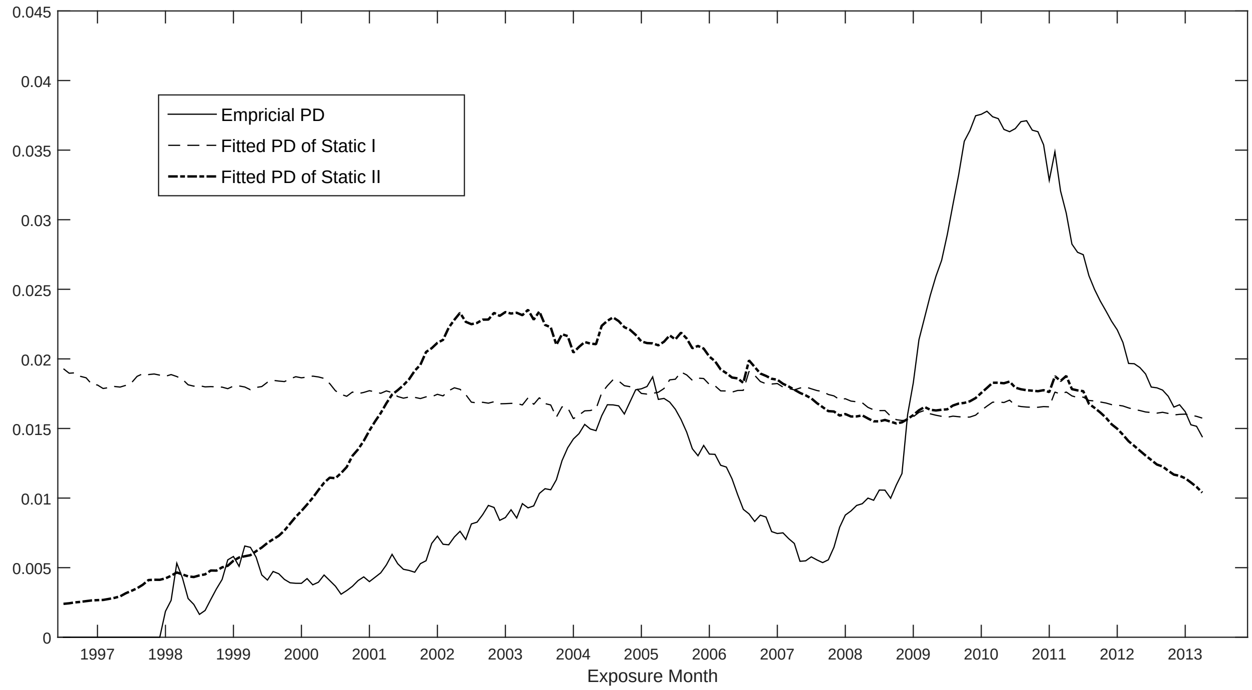
Notes: The picture shows empirical default rates of mortgages grouped by mortgage age. The horizontal axis is mortgage age in months and the vertical axis is default rates.

Figure 2: Default Rates by Mortgage Age of Static I and Static II



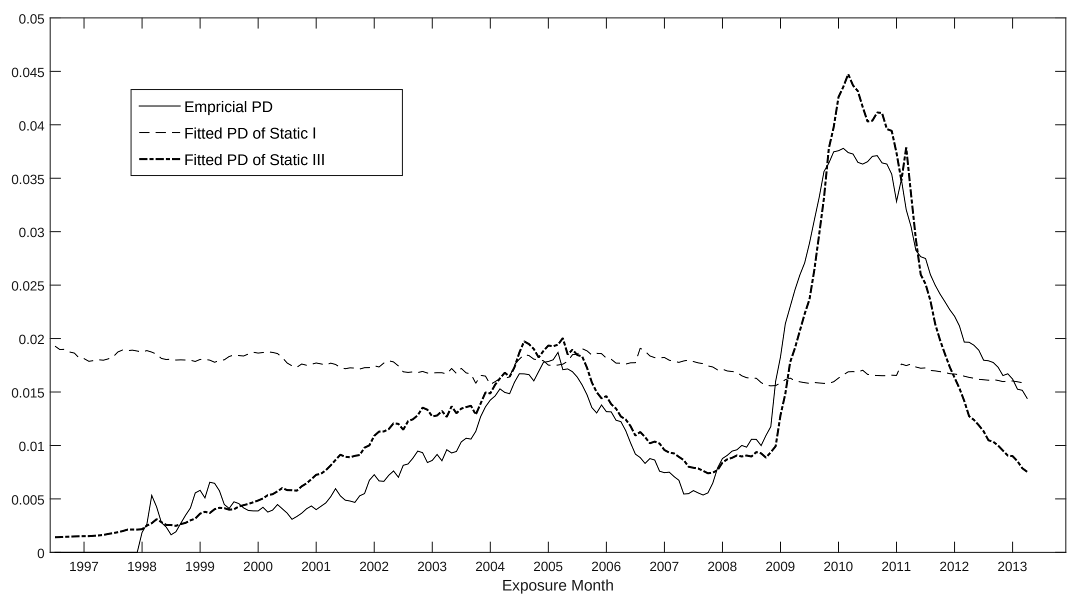
Notes: The horizontal axis is mortgage age in months. The solid line shows empirical default rates. The light dashed line represents fitted default rates of Static I, which uses mortgage age, DSCR and SW. The dark dashed line refers to fitted default rates of Static II. Variables of this model consist of mortgage age function, DSCR and SW.

Figure 3: Default Rates by Exposure Month of Static I and Static II



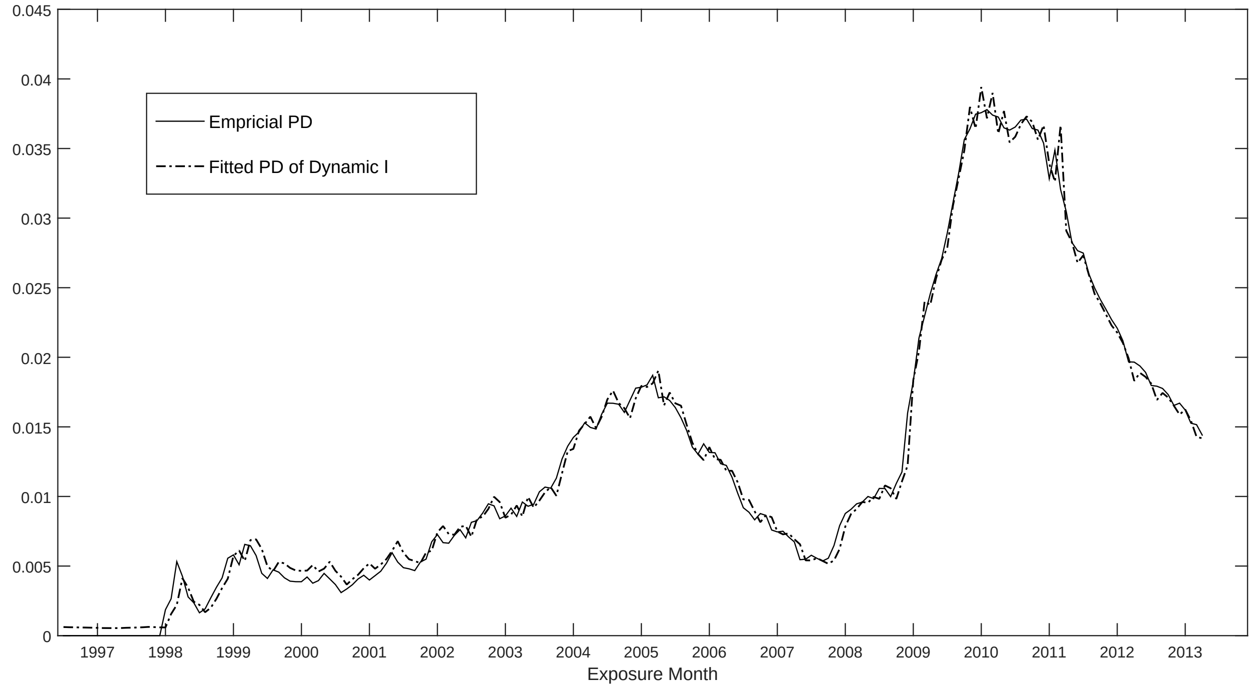
Notes: The picture reports default rates of mortgages grouped by exposure month. The horizontal axis is exposure month. The solid line shows empirical default rates. The light dashed line represents fitted default rates of Static I, which uses mortgage age, DSCR and SW. The dark dashed line refers to fitted default rates of Static II. Variables of this model consist of mortgage age function, DSCR and SW.

Figure 4: Default Rates by Exposure Month of Static I and Static III



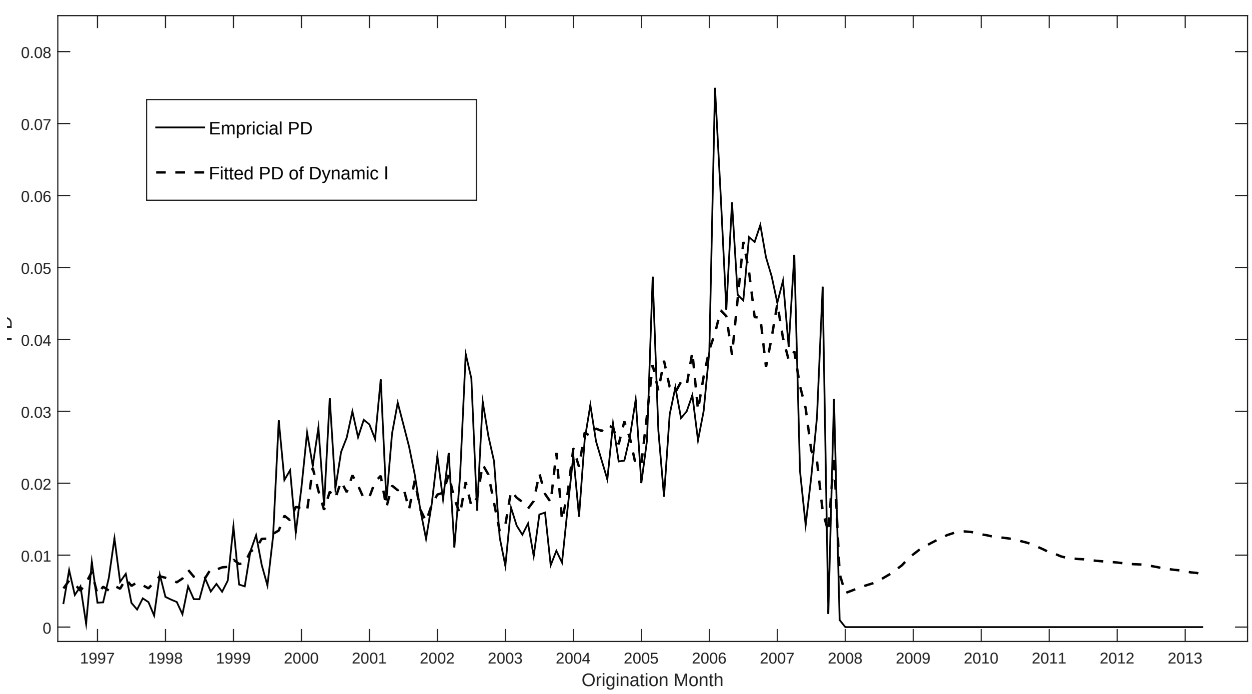
Notes: The picture reports default rates of mortgages grouped by exposure month. The horizontal axis is exposure month. The solid line shows empirical default rates. The light dashed line represents fitted default rates of static model one, which uses mortgage age, DSCR and SW. The dark dashed line refers to fitted default rates of static model three. Variables of this model consist of mortgage age function, DSCR, SW, and lagged_PD.

Figure 5: Default Rates by Exposure Month of Dynamic I



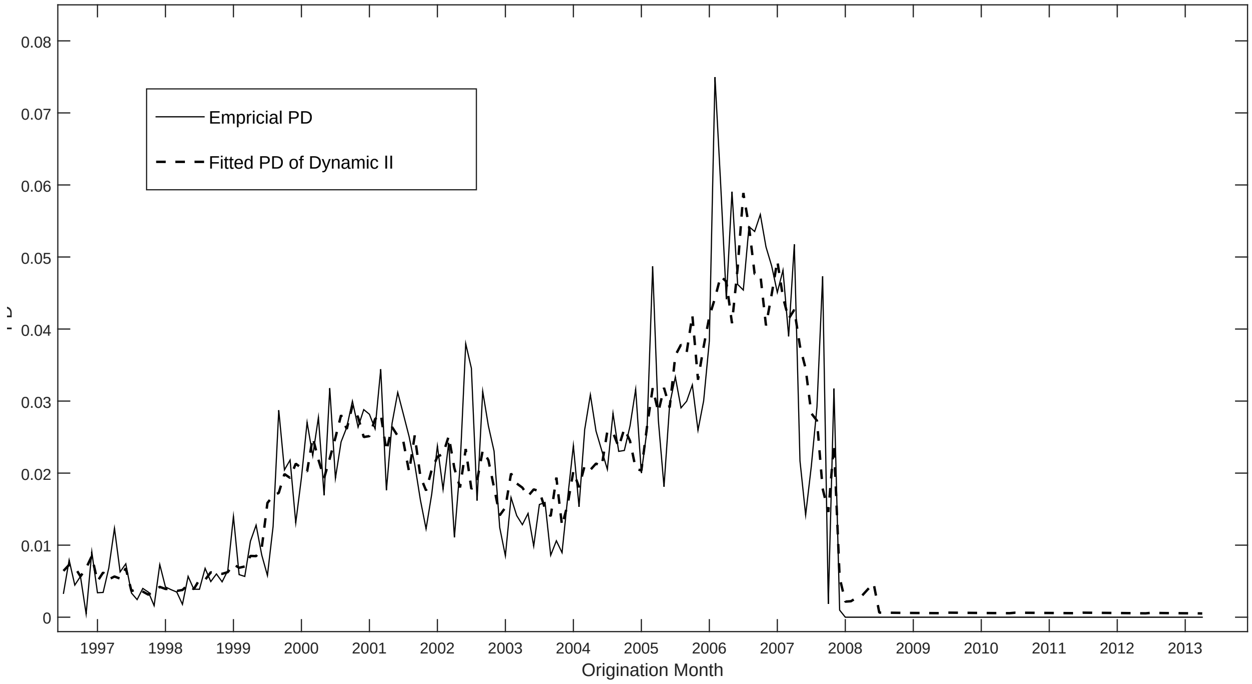
Notes: The picture reports default rates of mortgages grouped by exposure month. The horizontal axis is exposure month. The solid line shows empirical default rates. The dashed line represents fitted default rates of Dynamic I. Variables of this model include mortgage age function, DSCR, SW, lagged_PD, and single-frailty factor.

Figure 6: Default Rates by Origination Month of Dynamic I



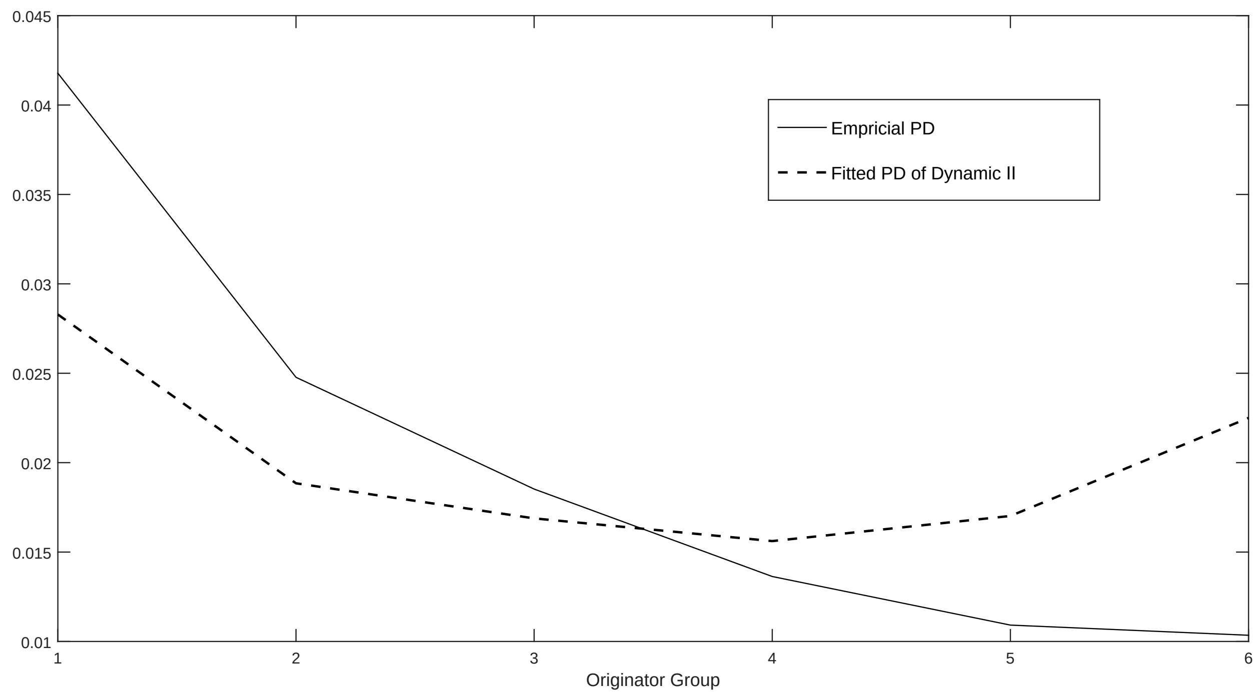
Notes: The picture reports default rates of mortgages grouped by origination month. The horizontal axis is origination month. The solid line shows empirical default rates. The dashed line represents fitted default rates of Dynamic I. Variables of this model include mortgage age function, DSCR, SW, lagged_PD, and single-frailty factor.

Figure 7: Default Rates by Origination Month of Dynamic II



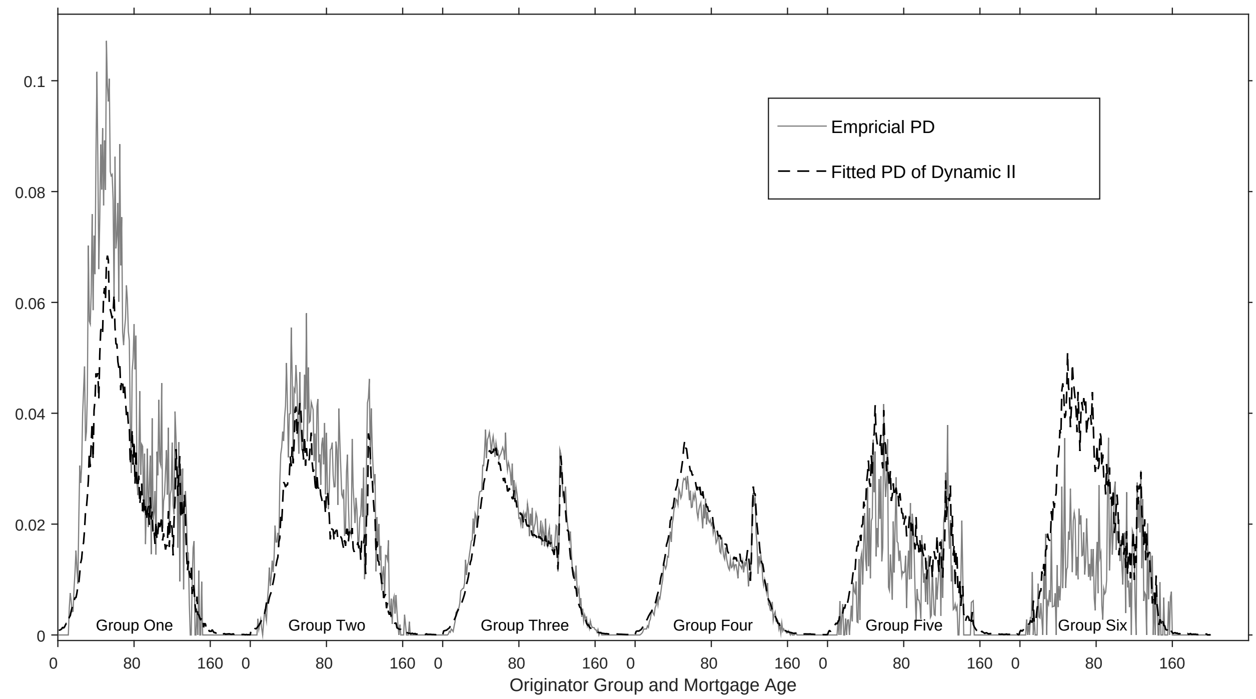
Notes: The picture reports default rates of mortgages grouped by origination month. The horizontal axis is origination month. The solid line shows empirical default rates. The dashed line represents fitted default rates of Dynamic II. Variables in this model include mortgage age function, DSCR, SW, lagged_PD, and origination month frailty factor.

Figure 8: Default Rates by Originator Group of Dynamic II



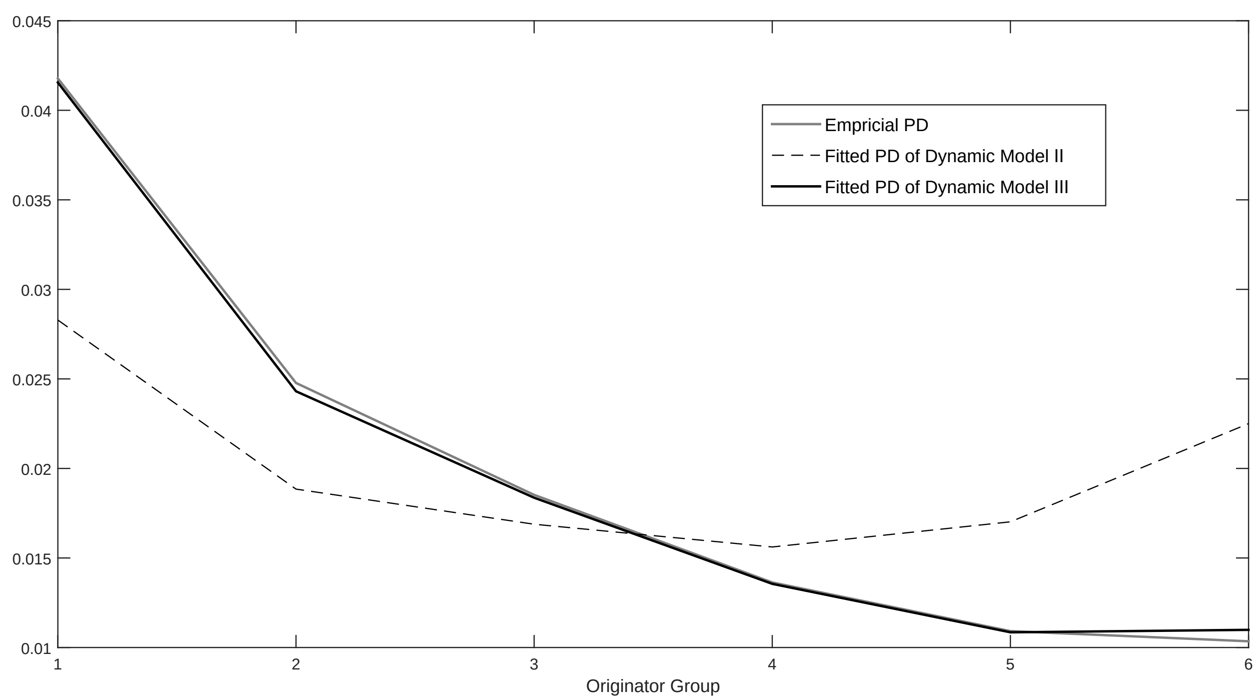
Notes: The picture reports default rates of mortgages grouped by originator. The horizontal axis in the figure is originator group. The solid line shows empirical default rates. The dashed line represents fitted default rates of Dynamic II. Variables in this model include mortgage age function, DSCR, SW, lagged_PD, and origination month frailty factor.

Figure 9: Default Rates by Originator Group and Mortgage Age of Dynamic II



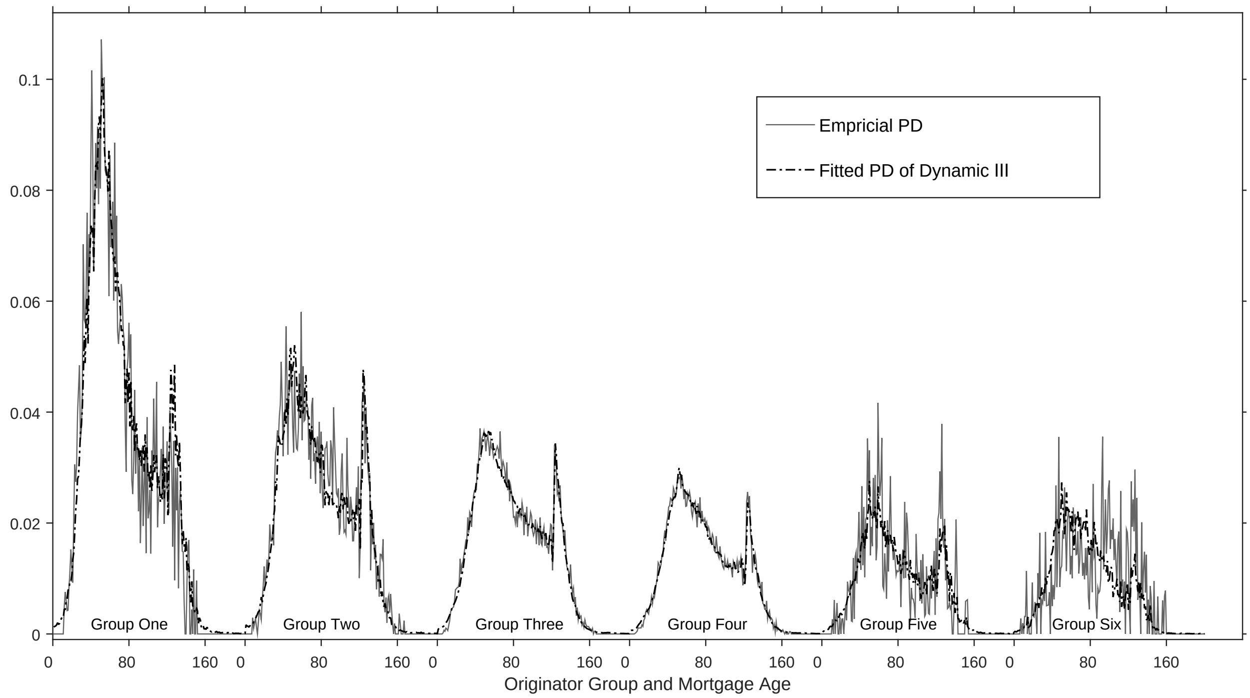
Notes: The picture reports default rates of mortgages grouped by originator and mortgage age. The horizontal axis in the figure is originator group and mortgage age. The light line shows empirical default rates. The dark line represents fitted default rates of Dynamic II. Variables in this model include mortgage age function, DSCR, SW, lagged_PD, and origination month frailty.

Figure 10: Default Rates by Originator Group of Dynamic II and Dynamic III



Notes: The picture reports default rates of mortgages grouped by originator. The horizontal axis is originator group. The light line shows empirical default rates. The dark line is fitted default rates of Dynamic III. Variables in this model include mortgage age function, DSCR, SW, lagged_PD, origination frailty factor, and originator frailty factor. The dashed line pertains to fitted default rates of Dynamic II, which include mortgage age function, DSCR, SW, lagged_PD, and origination month frailty factor.

Figure 11: Default Rates by Originator Group and Mortgage Age of Dynamic II and Dynamic III



Notes: The picture reports default rates of mortgages grouped by originator and mortgage. The horizontal axis is originator group and mortgage age. The light line shows empirical default rates. The dark line represents fitted default rates of Dynamic III. Variables in this model include mortgage age function, DSCR, SW, lagged_PD, origination frailty factor, and originator frailty factor.